Perfect 2-colorings of the generalized Petersen graph

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Abstract. In this paper, we enumerate the parameter matrices of all perfect 2-colorings of the generalized Petersen graphs $GP(n, 2)$, where $n \geq 5$. We also present some basic results for $GP(n, k)$, where $n \geq 5$ and $k \geq 3$.

Keywords. Perfect coloring; equitable partition; generalized Petersen graphs.

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1. Introduction

The concept of a perfect $m$-coloring plays an important role in graph theory, algebraic combinatorics, and coding theory (completely regular codes). There is another term for this concept in the literature as ‘equitable partition’ (see [7]).

The existence of completely regular codes in graphs is a historical problem in mathematics. Completely regular codes are a generalization of perfect codes. In 1973, Delsarte conjectured the non-existence of perfect codes in Johnson graphs. Therefore, some effort has been done on enumerating the parameter matrices of some Johnson graphs, including $J(6, 3)$, $J(7, 3)$, $J(8, 3)$, $J(8, 4)$, and $J(v, 3)$ ($v$ odd) (see [1, 2, 6]).

Fon-Der-Flaass enumerated the parameter matrices of $n$-dimensional hypercube $Q_n$ for $n < 24$. He also obtained some constructions and a necessary condition for the existence of perfect 2-colorings of the $n$-dimensional hypercube with a given parameter matrix (see [3–5]).

In this article we enumerate the parameter matrices of all perfect 2-colorings of $GP(n, 2)$.

2. Definition and concepts

In this section, we give some basic definitions and concepts.

DEFINITION 2.1

The generalized Petersen graph $GP(n, k)$ has vertices, respectively, edges given by

$V(GP(n, k)) = \{a_i, b_i : 0 \leq i \leq n - 1\}$,

$E(GP(n, k)) = \{a_{i-1}a_i, a_ib_i, b_ib_{i+k} : 0 \leq i \leq n - 1\}$,

where the subscripts are expressed as integers modulo $n$ ($\geq 5$), and $k \geq 1$ is the ‘skip’.

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